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SEQUENTIAL ESTIMATION OF BIOLOGICAL POPULATION IN THE FIELD

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SUMMARY

It is proposed to devise a sequential method of estimation of a biological population which is alive and freely moving. Assuming that the probability is proportional to the size of a sub-region : a well-defined region of a biological population of a given class, an estimator has been derived on the basis of observed distances between individuals and their nearest neighbours. A comparative study has been made with the classical and maximum likelihood estimators and to test the veracity of the method, an empirical study has been made.

Keywords : Biological population; Sequential method; Maximum Likelihood Estimator.

Introduction

Baily [2] and Chapman [3] used some sequential methods for estimating the size of a biological population by capture and recapture method. Anscombe [1] discussed sequential estimation at length and made a comparative study with classical fixed size method. Singh [5] discussed some biased types of estimators and Singh and Singh [6] and Chaudhary and Khatri [4] have proposed a new line of estimation for biologists in the light of a well-defined sampling structure. These methods do not suffice the purpose for estimating the biological population in the field and the method of capture, recapture cannot be applied. In this paper, it is proposed to derive a sequential method of estimation of a biological population which is based on observed distances between individuals and their nearest neighbours with the assumption that the population is alive and freely moving.

Let the size of population be N which is finite but unknown. If the ratio of the size of subregion chosen to that of whole region is p and if n is the number found in the sub-region, the probability that a number of a biological population of a given class in any sub-region of a well-defined region is proportional to the size of the sub-region, it may be written as:

$$P(n) = \binom{N}{n} p^n (1 - p)^{N-n} \tag{1.1}$$

Which gives the maximum likelihood estimator $\hat{N}$ satisfying the inequality :

$$\frac{n}{p} - 1 \leqslant \hat{N} \leqslant \frac{n}{p} \tag{1.2}$$

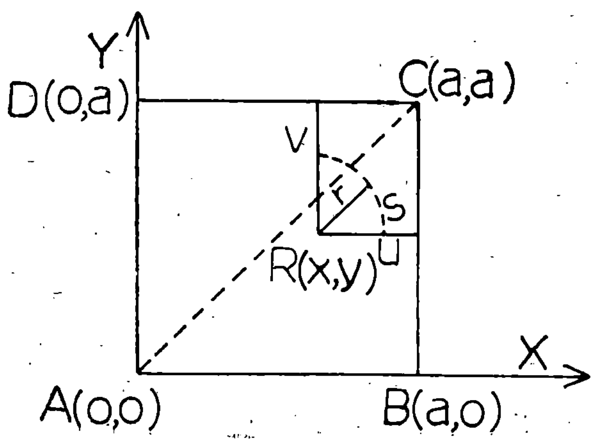
If the sample size is sufficiently large the variance of (1.2) can be written as :

$$V(\hat{N}) = N(1 - p)/p \tag{1.3}$$

Let us ellaborate this idea of area ratio in terms of the distance observed between two nearest individuals in the given sub-region.

2. Area with Defined/Confined Boundaries of the Field

Let the region be a plane area and let the coordinates of R of an individual chosen at random be (x, y), which are distributed independently and with rectangular law over the area with the assumption that the nearest individuals of the population be at the distance r.



Without any loss of generality, it can be assumed that the region be a square with its vertices $A(0, 0)$, $B(a, 0)$, $C(a, a)$ and $D(0, a)$. Since the population under consideration is alive and freely moving at any rate such that every random choice of R provides an independent value of r. There can arise four situations :

(i) $r \leqslant$ the smaller of u or v
(ii) $r >$ the smaller of u or v, but $\leqslant$ the larger
(iii) $r >$ the larger u and v, but $\leqslant (u^2 + v^2)^{1/2}$
(iv) $r > (u^2 + v^2)^{1/2}$

Where,

$$u = a - x \text{ and } v = a - y.$$

For case (i) suppose that R lies below (a) AC or (b) BD. Let us suppose that R lies below AC for which $r \leqslant u$. The probability density for r is :

$$(Nc_1) \frac{2\pi\, r/4}{a^2}\left(1 - \frac{\pi r^2}{4a^2}\right)^{N-1} dr \tag{2.1}$$

for $r \leqslant u$. Probability density for R is given by :

$$Nc_1 \frac{\pi r}{2a^2}\left(1 - \frac{\pi r^2}{4a^2}\right)^{N-1}\left(1 - \frac{r}{a}\right)^2 dr \quad \text{for } r \leqslant u \tag{2.2}$$

Hence the likelihood for this case may be written as :

$$L = \prod^{m} N\frac{\pi r}{2a^2}\left(1 - \frac{\pi r^2}{4a^2}\right)^{N-1}\left(1 - \frac{r}{a}\right)^2 \tag{2.3}$$

From (2.3) the maximum likelihood estimator of N is obtained as :

$$\hat{N} = -m\Big/\sum \text{Log}\left(1 - \frac{\pi r^2}{4a^2}\right) \tag{2.4}$$

with variance

$$V(\hat{N}) \cong \pi N^2/m \tag{2.5}$$

3. Area without Defined/Confined Boundaries

Let us assume that the region be a circular with centre as origin and radius a. Here three situations can arise :

(i) $r < \sqrt{a^2 - (x^2 + y^2)}$
(ii) $r \leqslant$ the smaller of u or v
(iii) $r >$ the smaller of u or v, but less than larger.

For case (i) suppose that R lies above the point $(a, 0)$. The probability density for r can be written as :

$$N c_1 \frac{2r}{a^2}\left(1-\frac{r^2}{a^2}\right)^{N-1} dr \tag{3.1}$$

Therefore, the probability density for R is given by :

$$\frac{2Nr}{a^2}\left(1-\frac{r^2}{a^2}\right)^{N-1}\left(1-\frac{r}{a}\right)^2 dr \tag{3.2}$$

Hence the likelihood for this case may be written as :

$$L=\prod^{m} N \frac{\pi r}{2a^2}\left(1-\frac{r^2}{a^2}\right)^{N-1}\left(1-\frac{r}{a}\right)^2 \tag{3.3}$$

From (3.3) the maximum likelihood estimator of N can be obtained as :

$$\hat{N}=-m\Big/\sum \log\left(1-\frac{r^2}{a^2}\right) \tag{3.4}$$

with variance $V(\hat{N}) \cong \dfrac{N^2}{m}$ (3.5)

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